Texture Zero Mass Matrices And Their Implications

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Introduction

- Understanding fermion masses and mixings constitutes one of the most outstanding problems of the present day High Energy Physics.

- One has a fairly good idea of the quark masses and mixing angles.

$$m_u, m_d \sim \text{few MeV}, m_s \sim 55 \text{ MeV}, m_c \sim 0.6 \text{ GeV},$$

$$m_b \sim 2.9 \text{ GeV}, m_t \sim 170\text{GeV}. \qquad (\text{at } M_Z \text{ scale})$$

$$s_{12} \sim 0.22, \quad s_{23} \sim 0.04, \quad s_{13} \sim 0.0035$$

- Interestingly, masses and mixing angles exhibit a clear cut hierarchy.

- The recent refinements of the angle θ_{13}, along with the precision measurement of θ_{12} and θ_{23} and the neutrino mass-squared differences, have given a new impetus to the neutrino oscillation phenomenology.

$$\Delta m^2_{solar} \sim 7.5 \times 10^{-5} \text{ eV}^2, \Delta m^2_{atm} \sim 2.5 \times 10^{-3} \text{ eV}^2,$$

$$s_{12} \sim 0.55, \quad s_{23} \sim 0.61, \quad s_{13} \sim 0.15$$

- This has further been strengthened by a recent constraint on the sum of absolute neutrino masses provided by the Planck experiment.

$\Sigma m_\nu < 0.23$ eV at 95 % C.L.

- The patterns of quark and lepton mixing angles are significantly different.

- The direct neutrino mass experiments based on β decay of specific isotopes (^3H, ^{187}Re, ^{163}Ho) obtain an upper bound~2 eV on the lightest neutrino mass. Expected to be sharpened by an order of magnitude by KATRIN.

- Similarly, for the effective Majorana mass m_{ee} experimental data has been able to provide an upper bound $m_{ee} < 0.1 – 0.25$ eV. Expected to be redefined largely by several next generation NDBD experiments aiming to achieve a sensitivity up to 0.01 eV in the near future.

Essentials pertaining to texture zero mass matrices

- In order to decipher the mystery of fermion masses and flavor mixings, various attempts have been made on the experimental as well as phenomenological front.

- On the experimental side, continuous refinements are being carried out in the quark as well as lepton mixing data which urge for a deeper understanding of several aspects of flavor physics.

- The present-day phenomenological approaches can be broadly categorized as "top-down" and "bottom-up".

- The top-down approach essentially starts with the formulation of mass matrices at the GUT scale, whereas, the bottom-up approach starts with the phenomenological mass matrices at the weak scale.

- In the present context, we follow the bottom-up approach. A successful phenomenological formulation of mass matrices may provide clues for appropriate dynamical models, in particular, important clues for their formulation through the "top-down" approach.

- The fermion masses and mixings are encoded in the couplings of Higgs and fermions.

$$\mathcal{L}_{\text{Yukawa}} = \sum_{j,k=1}^{3} (Y_{jk} \overline{(q,q')_{jL}} \begin{pmatrix} h^{(0)*} \\ -h^{-} \end{pmatrix} q_{kR} + Y'_{jk} \overline{(q,q')_{jL}} \begin{pmatrix} h^{+} \\ h^{0} \end{pmatrix} q'_{kR} + h.c.$$

- Mass matrices are related to Yukawa couplings as

$$M_U = -\frac{v}{\sqrt{2}} Y_{jk}, \; M_D = -\frac{v}{\sqrt{2}} Y'_{jk}.$$

- These fermion mass matrices are arbitrary 3X3 complex matrices containing, in general, a total of 36 real free parameters.

- For these matrices to provide valuable clues for developing an understanding of flavor physics, it is desirable that the number of free parameters of these is constrained.

- Without loss of generality, in the SM and its extensions in which right-handed quarks are singlets, fermion mass matrices can be considered to be Hermitian.

- These Hermitian matrices are characterized by 18 free parameters that are still large in number as compared to the observables.

- To further reduce the number of free parameters, Fritzsch (PLB 1977, 1978) had initiated the first step by proposing the earliest ansatze made in the context of quark mass matrices, essentially laying down the path for future investigations in this direction.

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & 0 & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & 0 & B_D \\ 0 & B_D^* & C_D \end{pmatrix}.$$

- A particular texture structure is said to be texture n zero, if it has n number of non-trivial zeros, i.e., if the sum of the number of diagonal zeros and half the number of the symmetrically placed off diagonal zeros is n.

- On the lines of these ansatze, texture zero mass matrices are formulated wherein certain elements of these are assumed to be zero, usually referred to as texture zeros.

- The non Fritzsch-like mass matrices differ from the above mentioned Fritzsch-like mass matrices in regard to the position of "zeros" in the structure of the mass matrices. One can get non Fritzsch-like mass matrices by shifting the position of Ci (i= U,D) on the diagonal as well as by shifting the position of zeros among the non diagonal elements.

- This leads to a large number of possible texture zero mass matrices. For example, for the case of texture 4 zero mass matrices, the number of possible matrices is 24 X 24=576.

- The viability of the formulated mass matrices is ensured by examining the compatibility of the mixing matrices so obtained from these with the low energy data.

- A large number of analyses over the past few years have established the texture zero approach as a viable framework for explaining the fermion masses and mixing data in the quark as well as lepton sector.

  - **H. Fritzsch and Z. Z. Xing, *Prog. Part. Nucl. Phys.*(2000).**
  - **Z. Z. Xing and H. Zhang, *J. Phys. G* (2004).**
  - **M. Gupta and G. Ahuja, *Int. J. Mod. Phys. A* (2012).**
  - **S. Sharma, P. Fakay, G. Ahuja, M. Gupta, *Phys.Rev.D*(2015).**
  - **S. Sharma, G. Ahuja, M. Gupta, *Phys.Rev.D*(2016).**

- However, despite showing considerable promise, the limitation of the texture zero approach is that the number of possible mass matrices is very large, resulting in an exhaustive case-by-case analysis of all possible texture-zero mass matrices.

A brief outline of the Weak Basis (WB) transformations

- Within the framework of the SM and its extensions, one has the freedom to make unitary transformations, referred to as "weak basis (WB) transformations" W on the quark fields, e.g.,

$$q_L \to W q_L \ , \ q_R \to W q_R , \ \ q'_L \to W q'_L \ , \ q'_R \to W q'_R$$

under which the gauge currents

$$L_W = \frac{g}{\sqrt{2}} \overline{(u,c,t)_L} \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu + h.c.$$

remain real and diagonal but the mass matrices transform as

$$M_U \to W^\dagger M_U W \ , \ M_D \to W^\dagger M_D W .$$

- Using WB transformations, several attempts have been made wherein the above freedom is exploited to introduce zeros in the general quark mass matrices.

- WB transformation approach broadly leads to two kinds of structures of the mass matrices, the following investigated by Branco *et al.*

$$M_q = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \ M_{q'} = \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}, \qquad q, q' = U, D$$

  and the other by Fritzsch and Xing

$$M_q = \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \qquad q = U, D.$$

- Hermiticity of the fermion mass matrices remains preserved under such transformations. Also, it can be checked that the CKM matrix is independent of WB transformations.

- The structure of the mass matrices so obtained can thereafter be considered texture specific and same methodology can be used to analyze these.

- Interestingly, now the large number of possible structures are not all independent. Several of these are related through WB transformations and therefore yield the same structure of the diagonalizing transformations leading to similar mixing matrices, making the number of matrices to be analyzed much less than before.

Texture zero quark mass matrices

S. Sharma, P.Fakay, G. Ahuja, M.Gupta, PRD (2015)

- Starting with the most general mass matrices, we have made an attempt to explore the possibility of obtaining a finite set of viable texture zero mass matrices formulated by invoking weak basis transformations.
- Following Hermitian mass matrices can be considered to be the most general ones.

$$M_q = \begin{pmatrix} E_q & A_q & F_q \\ A_q^* & D_q & B_U \\ F_q^* & B_q^* & C_q \end{pmatrix} \qquad (q = U, D),$$

- The concept of WB transformations introduces zeros in these matrices using a unitary matrix W

$$M_U = \begin{pmatrix} E_U & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} E_D & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}.$$

- For M_U and M_D, instead of zeros being in the (1,3) and (3,1) positions, these could also be in either the (1,2) and (2,1) or (2,3) and (3,2) position. These other structures are related to the above mentioned matrices as we have the facility of subjecting M_U and M_D to another WB transformation that can be the permutation matrix P.

- These different mass matrices, however, yield the same CKM matrix, therefore while presenting the results of the analysis, it is sufficient to discuss any one of these matrices.

- In order to further constrain the parameter space available to the elements of these mass matrices, one can consider the following hierarchy for the elements of the matrices.

$$(1,i) \lesssim (2,j) \lesssim (3,3) ; \quad i = 1,2,3 , \quad j = 2,3 .$$

- After obtaining these matrices, as a next step, their viability needs to be ensured by examining the compatibility of the CKM matrix reproduced through these mass matrices with the recent quark mixing data.

- In order to reproduce the CKM matrix, one needs to diagonalize the mass matrices.

- To facilitate diagonalization, for q =U, D, the mass matrix M_q may be expressed as $M_q = Q^{\dagger}_q M^r_q Q_q$, implying $M^r_q = Q_q M_q Q^{\dagger}_q$ where M^r_q is a symmetric matrix with real eigenvalues and Q_q is the diagonal phase matrix.

$$M_q^r = \begin{pmatrix} E_q & |A_q| & 0 \\ |A_q| & D_q & |B_q| \\ 0 & |B_q| & C_q \end{pmatrix}, \qquad Q_q = \begin{pmatrix} e^{-i\alpha_q} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\beta_q} \end{pmatrix}.$$

The phases of the mass matrices can be related through the parameters Φ_1 and Φ_2 as $\Phi_1 = \alpha_U - \alpha_D$ and $\Phi_2 = \beta_U - \beta_D$.

- The matrix M^r_q can be diagonalized using the transformations

$$M_q^{\text{diag}} = O_q^T M_q^r O_q = O_q^T Q_q M_q Q_q^\dagger O_q = \text{Diag}(m_1, -m_2, m_3)$$

where the subscripts 1, 2, and 3 refer, respectively, to u, c, t for the up sector and d, s, b for the down sector.

- The exact diagonalizing transformation O_q for the matrix M^r_q is given by

$$O_q = \begin{pmatrix} \sqrt{\frac{(E_q+m_2)(m_3-E_q)(C_q-m_1)}{(C_q-E_q)(m_3-m_1)(m_2+m_1)}} & \sqrt{\frac{(m_1-E_q)(m_3-E_q)(C_q+m_2)}{(C_q-E_q)(m_3+m_2)(m_2+m_1)}} & \sqrt{\frac{(m_1-E_q)(E_q+m_2)(m_3-C_q)}{(C_q-E_q)(m_3+m_2)(m_3-m_1)}} \\ \sqrt{\frac{(C_q-m_1)(m_1-E_q)}{(m_3-m_1)(m_2+m_1)}} & -\sqrt{\frac{(E_q+m_2)(C_q+m_2)}{(m_3+m_2)(m_2+m_1)}} & \sqrt{\frac{(m_3-E_q)(m_3-C_q)}{(m_3+m_2)(m_3-m_1)}} \\ -\sqrt{\frac{(m_1-E_q)(m_3-C_q)(C_q+m_2)}{(C_q-E_q)(m_3-m_1)(m_2+m_1)}} & \sqrt{\frac{(E_q+m_2)(C_q-m_1)(m_3-C_q)}{(C_q-E_q)(m_3+m_2)(m_2+m_1)}} & \sqrt{\frac{(m_3-E_q)(C_q-m_1)(C_q+m_2)}{(C_q-E_q)(m_3+m_2)(m_3-m_1)}} \end{pmatrix}.$$

- These diagonalizing transformations are related to the mixing matrix as

$$V_{\text{CKM}} = O_U^T Q_U Q_D^\dagger O_D.$$

Numerical analysis

- The quark masses and mass ratios at M_Z scale are

$$m_u = 1.38^{+0.42}_{-0.41}\,\text{MeV}, \quad m_d = 2.82 \pm 0.48\,\text{MeV}, \quad m_s = 57^{+18}_{-12}\,\text{MeV},$$

$$m_c = 0.638^{+0.043}_{-0.084}\,\text{GeV}, \; m_b = 2.86^{+0.16}_{-0.06}\,\text{GeV}, \; m_t = 172.1 \pm 1.2\,\text{GeV},$$

$$m_u/m_d = 0.553 \pm 0.043, m_s/m_d = 18.9 \pm 0.8.$$

- Values of quark mixing parameters are:

$$|V_{us}| = 0.22534 \pm 0.00065, \; |V_{ub}| = 0.00351^{+0.00015}_{-0.00014}, \; |V_{cb}| = 0.0412^{+0.0011}_{-0.0005},$$

$$\text{Sin}2\beta = 0.679 \pm 0.020.$$

- The parameters$\Phi_1$and $\Phi_2$related to the phases of the mass matrices have been given full variation from 0 to 2π. The free parameters E_U, E_D, D_U and D_D have also been given wide variation in conformity with the condition imposed on hierarchy of the elements of the mass matrices as well as to ensure that the elements of orthogonal diagonalizing transformations should remain real.

- Using the relation between mass matrices and mixing matrix, the resultant CKM matrix comes out to be

$$V_{CKM} = \begin{pmatrix} 0.9739-0.9745 & 0.2246-0.2259 & 0.00337-0.00365 \\ 0.2224-0.2259 & 0.9730-0.9990 & 0.0408-0.0422 \\ 0.0076-0.0101 & 0.0408-0.0422 & 0.9990-0.9999 \end{pmatrix}$$

- Fully compatible with the one given by PDG.

- Also, the CP-violating Jarlskog's rephasing invariant parameter J comes out to be $(2.494 - 3.365)\times 10^{-5}$, which again is compatible with its latest experimental range, $(2.96^{+0.20}_{-0.16})\times 10^{-5}$.

- The above-mentioned compatibility leads to the viability of the general mass matrices.

- As a next step, noting that the number of free parameters associated with these matrices is larger than the number of observables, it becomes interesting to examine whether any of their elements is redundant.

- To this end, we present below the magnitudes of the elements of matrices M_U and M_D

$$M_U = \begin{pmatrix} 0-0.00138 & 0.006-0.042 & 0 \\ 0.006-0.042 & 26.46-102.68 & 62.82-86.10 \\ 0 & 62.82-86.10 & 68.78-145.00 \end{pmatrix} \text{GeV},$$

$$M_D = \begin{pmatrix} 0-0.00127 & 0.011-0.019 & 0 \\ 0.011-0.019 & 0.36-1.66 & 1.03-1.44 \\ 0 & 1.03-1.44 & 1.16-2.44 \end{pmatrix} \text{GeV}.$$

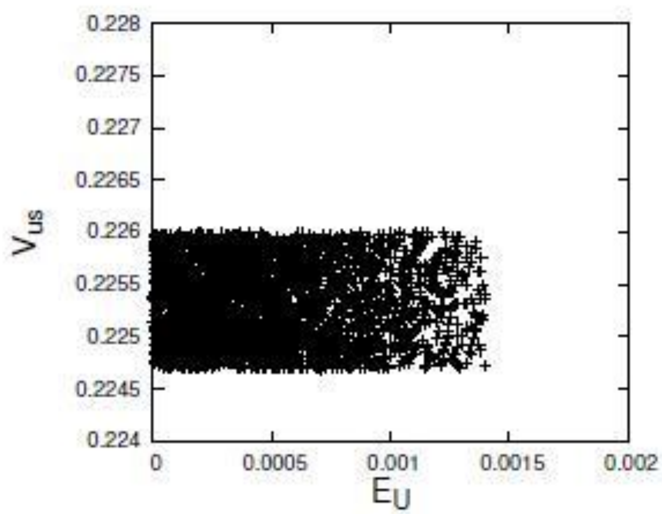
- Interestingly, the above matrices reveal that their (1,1) element (E_U, E_D) is quite small in comparison with the other non-zero elements. This, in turn, implies that E_U and E_D of the matrices M_U and M_D, respectively, can be ignored altogether without loss of parameter space.

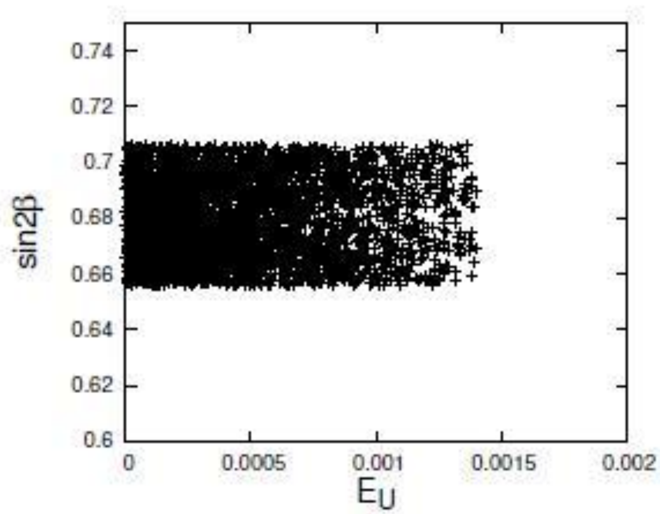
- To confirm this, one should examine the effect of the variation of these on CKM parameters.

- The parameter E_U assumes quite small values < 0.0014 GeV.

  Also both V_{us} and Sin2β seem independent of the range of E_U, indicating the redundancy of element E_U.

- Similar conclusions can be drawn from E_D versus the CKM matrix elements plot.





- Ignoring the elements E_U and E_D of the mass matrices, one gets M_U and M_D as

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}$$

  indicating a transition from texture 2 zero mass matrices to texture 4 zero mass matrices.

- Carrying out a similar analysis for these matrices, the corresponding CKM matrix comes out to be

$$V_{CKM} = \begin{pmatrix} 0.9741-0.9744 & 0.2246-0.2259 & 0.00337-0.00365 \\ 0.2245-0.2258 & 0.9732-0.9736 & 0.0407-0.0422 \\ 0.0071-0.0100 & 0.0396-0.0417 & 0.9990-0.9992 \end{pmatrix}.$$

- This matrix is in agreement with the latest quark mixing matrix given by PDG and is also fully compatible with the CKM matrix constructed earlier.

- Further, the range of the parameter J comes out to be again compatible with its latest experimental range, justifying our earlier conclusion that the elements E_U and E_D are essentially redundant.

- Apart from the form of texture 4 zero mass matrices considered above, there are several other possible texture 4 zero structures. Based on whether the matrices are related through permutations or not, all possible texture 4 zero mass matrices can be classified as shown in Table.

	a	b	c	d	e	f
Category 1	$\begin{pmatrix} \mathbf{0} & A & \mathbf{0} \\ A^* & D & B \\ \mathbf{0} & B^* & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & A \\ \mathbf{0} & C & B \\ A^* & B^* & D \end{pmatrix}$	$\begin{pmatrix} D & A & B \\ A^* & \mathbf{0} & \mathbf{0} \\ B^* & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} C & B & \mathbf{0} \\ B^* & D & A \\ \mathbf{0} & A^* & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} D & B & A \\ B^* & C & \mathbf{0} \\ A^* & \mathbf{0} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & B \\ \mathbf{0} & \mathbf{0} & A \\ B^* & A^* & D \end{pmatrix}$
Category 2	$\begin{pmatrix} D & A & \mathbf{0} \\ A^* & \mathbf{0} & B \\ \mathbf{0} & B^* & C \end{pmatrix}$	$\begin{pmatrix} D & \mathbf{0} & A \\ \mathbf{0} & C & B \\ A^* & B^* & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & A & B \\ A^* & D & \mathbf{0} \\ B & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} C & B & \mathbf{0} \\ B^* & \mathbf{0} & A \\ \mathbf{0} & A^* & D \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & B \\ \mathbf{0} & D & A \\ B^* & A^* & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & B & A \\ B^* & C & \mathbf{0} \\ A^* & \mathbf{0} & D \end{pmatrix}$
Category 3	$\begin{pmatrix} \mathbf{0} & A & D \\ A^* & \mathbf{0} & B \\ D^* & B^* & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & D & A \\ D^* & C & B \\ A^* & B & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & A & B \\ A^* & \mathbf{0} & D \\ B^* & D^* & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & B & C \\ B^* & \mathbf{0} & A \\ C^* & A^* & D \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & C & B \\ C^* & D & A \\ B^* & A^* & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & B & A \\ B^* & \mathbf{0} & C \\ A^* & C^* & D \end{pmatrix}$
Category 4	$\begin{pmatrix} A & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & D & B \\ \mathbf{0} & B^* & C \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & B \\ \mathbf{0} & A & \mathbf{0} \\ B^* & \mathbf{0} & D \end{pmatrix}$	$\begin{pmatrix} C & B & \mathbf{0} \\ B^* & D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A \end{pmatrix}$	$\begin{pmatrix} A & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C & B \\ \mathbf{0} & B^* & D \end{pmatrix}$	$\begin{pmatrix} D & \mathbf{0} & B \\ \mathbf{0} & A & \mathbf{0} \\ B^* & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} C & B & \mathbf{0} \\ B^* & D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A \end{pmatrix}$

- For the matrices belonging to category 1, considering both M_U and M_D as 1a type, we have already shown that these are viable and explain the quark mixing data quite well. The other matrices of this category, related through permutation matrix, also yield similar results.

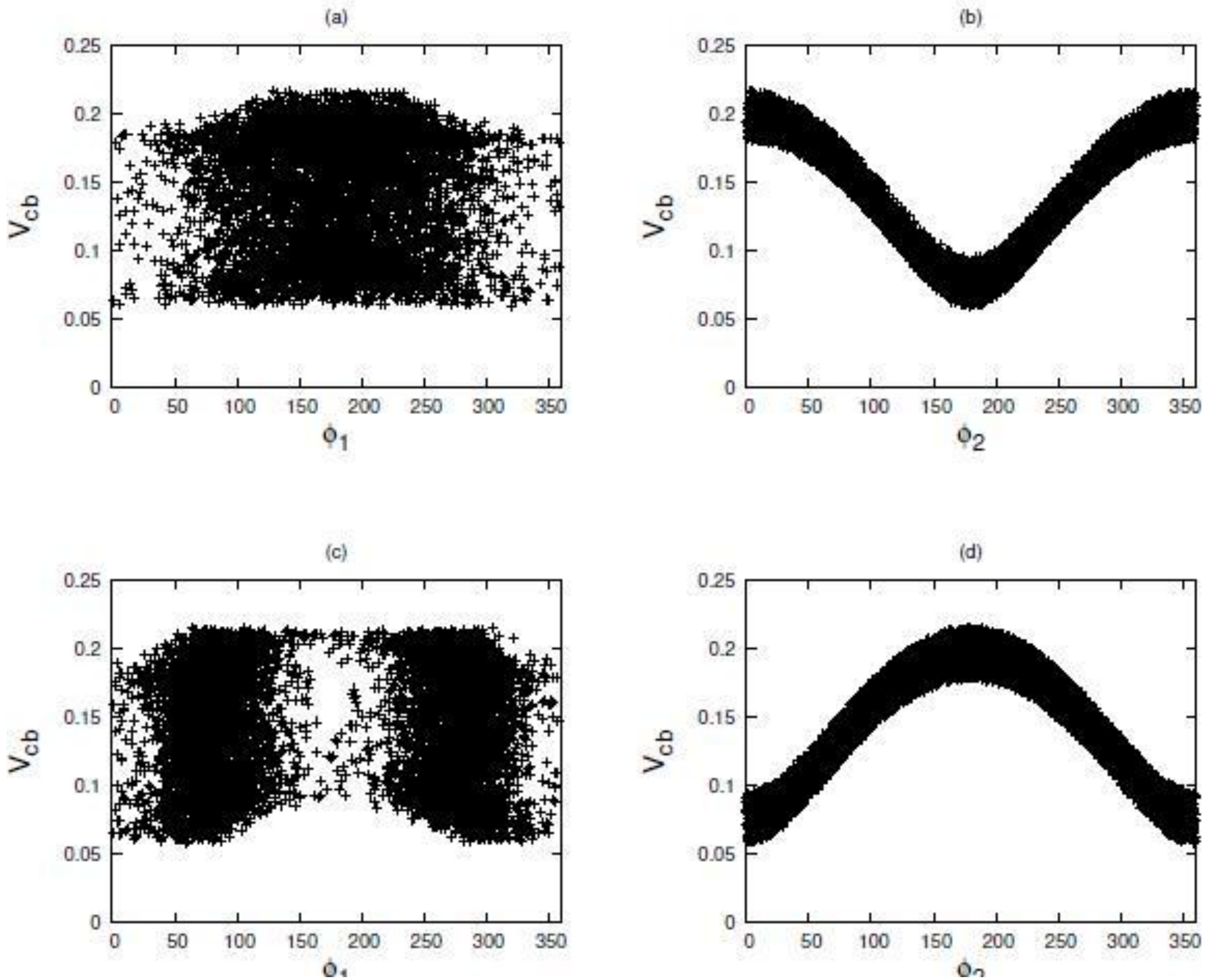
- For the matrices belonging to category 4, one finds that interestingly these are not viable as in all these matrices one of the generations gets decoupled from the other two.

- Further, for categories 2 and 3, again a similar analysis reveals that the matrices of these classes are also not viable as can be understood from the following CKM matrices obtained for categories 2 and 3 respectively, e.g.,

$$V_{CKM} = \begin{pmatrix} 0.9740-0.9744 & 0.2247-0.2260 & 0.0024-0.0099 \\ 0.2205-0.2256 & 0.9509-0.9727 & 0.0596-0.2172 \\ 0.0140-0.0445 & 0.0584-0.2127 & 0.9905-1.0000 \end{pmatrix}$$

$$V_{CKM} = \begin{pmatrix} 0.9736-0.9744 & 0.2247-0.2260 & 0.0098-0.0331 \\ 0.2226-0.2278 & 0.9549-0.9719 & 0.0659-0.1937 \\ 0.00007-0.0340 & 0.0694-0.1928 & 0.9810-0.9976 \end{pmatrix}$$

- Can be further verified from graphs plotted for categories 2 and 3 respectively.



- The plotted values of element V_{cb} have no overlap with its experimental range, therefore, these matrices can be considered to be non viable.

- The above discussion clearly brings out that only the texture 4 zero quark mass matrices belonging to category 1 of the table are found to be viable.

- Interestingly, the matrices considered here are quite similar to the original Fritzsch ansatze, except for their (2,2) element being non zero for both M_U and M_D.

Implications of the CP asymmetry parameter Sin2β

- After having arrived at a finite set of viable quark mass matrices, it is desirable to examine the implication of the precisely known parameter Sin2β, characterizing CP asymmetry $a_{\psi}K_s$ in the $B_d^o \rightarrow \psi K_s$ decay, on these mass matrices.

- The parameterSin2βprovides vital clues to the structural features of texture zero mass matrices, comprising of hierarchy and phases of the elements of the mass matrices.

- Earlier, using the strong hierarchy of the elements of the mass matrices
  $A_i << B_i \sim D_i << C_i$
  and considering only one phase ϕ_1 of the mass matrices, yielded the following expression for the angle β:

$$\beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] = \arg\left[1 - \sqrt{\frac{m_u m_s}{m_c m_d}} e^{-i\phi_1}\right]$$

- Unfortunately, the value of Sin2βpredicted by this formula was in quite disagreement with the known value, thereby posing the question whether CP violation could be accommodated by texture 4 zero Fritzsch-like mass matrices.

- However, in order to understand the relation between the parameter sin2β and the mass matrices, one needs to re-express Sin2β without considering any assumptions regarding the structural features, i.e., the phases and the hierarchy of the elements of the mass matrices.

- The structure of the exact diagonalizing transformation can be simplified keeping in mind $m_3 >> m_2 >> m_1$ and $C_i >> m_1$. This approximation induces less than a fraction of a percentage error in the numerical results.

$$O_i = \begin{pmatrix} 1 & \zeta_{1i}\sqrt{\frac{m_1}{m_2}} & \frac{\zeta_{2i}}{\zeta_{3i}}\sqrt{\frac{m_1 m_2}{m_3^2}} \\ \zeta_{3i}\sqrt{\frac{m_1}{m_2}} & -\zeta_{1i}\zeta_{3i} & \zeta_{2i} \\ -\zeta_{1i}\zeta_{2i}\sqrt{\frac{m_1}{m_2}} & \zeta_{2i} & \zeta_{1i}\zeta_{3i} \end{pmatrix}$$

with

$$\zeta_{1i} = \sqrt{1 + \frac{m_2}{C_i}}, \quad \zeta_{2i} = \sqrt{1 - \frac{C_i}{m_3}}, \quad \zeta_{3i} = \sqrt{\frac{C_i}{m_3}}.$$

- Eventually, one can express β in terms of the quark masses as well as the phases of the quark mass matrix, e.g.,

$$\beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] = \arg\left[\left(1-\sqrt{\frac{m_u m_s}{m_c m_d}}e^{-i(\phi_1+\phi_2)}\right)\left(\frac{1-r_2 e^{i\phi_2}}{1-r_1 e^{i\phi_2}}\right)\right]$$

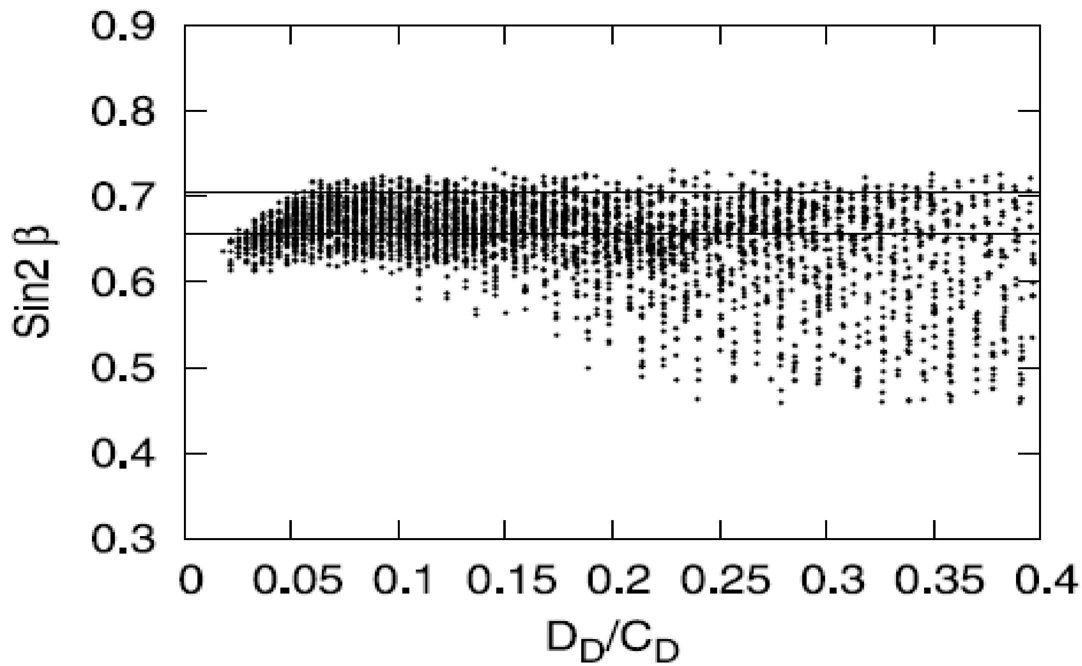
wherein

$$r_1 = \frac{\zeta_{1U}\,\zeta_{3U}\,\zeta_{1D}\,\zeta_{2D}}{\zeta_{2U}\,\zeta_{3D}} \qquad \text{and} \qquad r_2 = \frac{\zeta_{1U}\,\zeta_{3U}\,\zeta_{2D}}{\zeta_{2U}\,\zeta_{1D}\,\zeta_{3D}}.$$

- Using this, one gets a value of sin2βwhich agrees fully with the one obtained by PDG.

- In order to investigate the implications of the formula on the structural features of mass matrices, one can examine the constraints imposed on the ratio $D_i/C_i$for i=U,D, characterizing hierarchy, as well as on the phases of the mass matrices.

- To investigate the role of the hierarchy, we have plotted sin2β versus D_D/C_D.



- Several conclusions follow from the graph:

1. As we deviate from strong hierarchy, characterized by the ratio $D_D/C_D \sim 0.01$ towards weak hierarchy given by $D_D/C_D \geq 0.1$,we are able to reproduce the results.

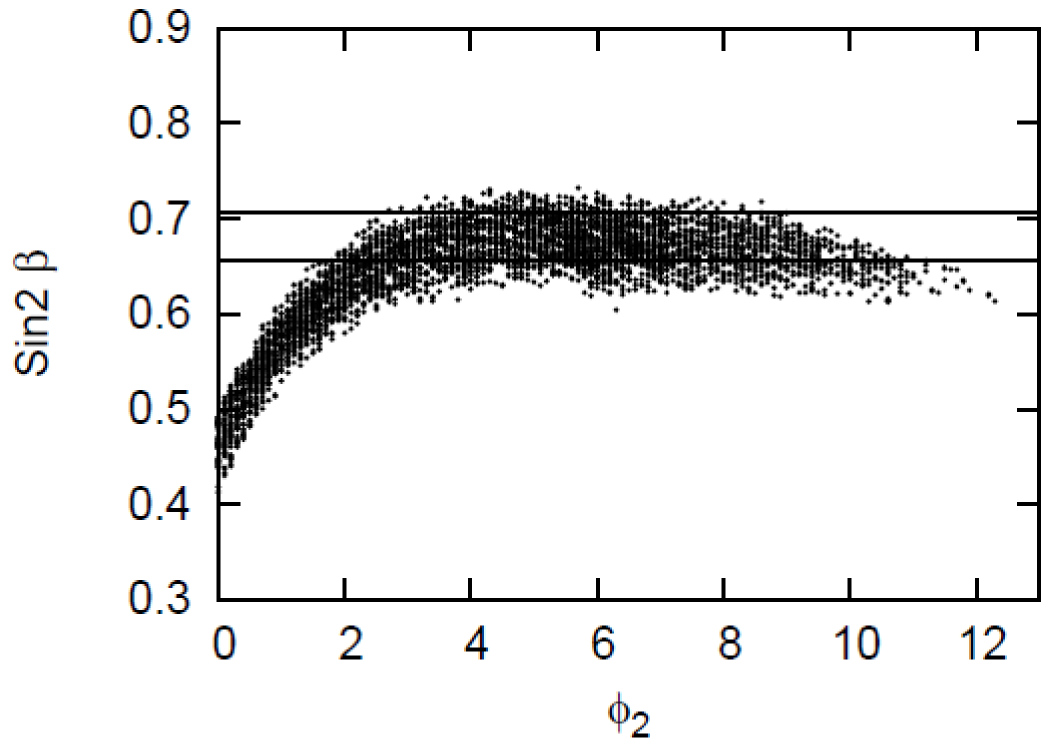
2. When $D_D/C_D \leq 0.03$, we are not able to reproduce any point within the 1σ range of $\sin 2\beta$.

3. However, only from $D_D/C_D \geq 0.05$, full range of $\sin 2\beta$ is reproduced.

- It is interesting to note that $\sin 2\beta$ as well as other CKM parameters can be reproduced even when $D_D/C_D \sim 0.6$, which suggests that the hierarchical mixing angles and masses can be reproduced by non hierarchical elements of the mass matrices.

- The corresponding graph of D_U/C_U is also very much similar.

- Coming to the issue of the phases of the mass matrices, a crucial role is played by the phase ϕ_2 of the mass matrices in fitting the data.



- For $\phi_2 = 0^\circ$, we are not able to reproduce sin2β despite giving full variation to other parameters.

- Further, in case one considers more zeros than the 4 texture zero mass matrices, we find that the present data rules out texture 5 and 6 zero quark mass matrices, confirming earlier conclusions in this regard.

- Therefore, in conclusion, the texture 4 zero quark mass matrices, similar to the original Fritzsch ansatze, and its permutations can be considered as a unique viable option compatible not only with the recent quark mixing data but also being able to well accommodate CP violation.

Texture zero lepton mass matrices

S. Sharma, G. Ahuja, M. Gupta, *Phys.Rev.D*(2016).

- Keeping in mind the analysis discussed in the case of quarks as well as the quark lepton universality, required by most of the Grand Unified Theories (GUTs),it becomes desirable to carry out a corresponding analysis in the leptonic sector as well.

- In the absence of any deep theoretical understanding of fermion masses and mixings, the 'smallness' of neutrino masses is best understood in terms of the Seesaw mechanism characterized by:

$$\mathbf{M_{\nu} = -\, M_{\nu D}^{T}\, M_{R}^{-1} M_{\nu D}}$$

with M_{ν}, $M_{\nu D}$ and M_R corresponding to the light Majorana neutrino mass matrix, the Dirac neutrino mass matrix and the heavy right handed Majorana neutrino mass matrix respectively.

- In the case of bottom-up approach for leptons, again good deal of emphasis has been on the texture zero approach, with most of the attempts made by considering the mass matrices to be in the ‘flavor basis’, wherein the charged lepton mass matrix M_l is considered to be diagonal while a texture is imposed on the Majorana neutrino matrix M_ν.

- Along with these, some attempts have also been carried out in the ‘non-flavor basis’ wherein it is usual to impose texture on the matrix M_l and on the Dirac neutrino mass matrix $M_{\nu D}$.

- For examining the viability of the mass matrices, the PMNS matrix is then obtained using the charged lepton mass matrix M_l and the Majorana neutrino matrix M_ν, the latter can be obtained through the seesaw relation by using $M_{\nu D}$ and the right handed Majorana neutrino mass matrix M_R.

- Within the framework of SM, the lepton mass matrices are completely arbitrary and are expressed in terms of complex 3X3 matrices.

- Within the SM and some of its extensions, without loss of parameter space, the general 3×3complex mass matrices M_l and $M_{\nu D}$ can be considered to be hermitian and in general expressed as

$$M_k = \begin{pmatrix} C_k & A_k & F_k \\ A_k^* & D_k & B_k \\ F_k^* & B_k^* & E_k \end{pmatrix} \qquad (k = l, \nu D).$$

- One can then use the facility of WB transformations, wherein it is possible to make a unitary transformation

$$\nu'_L = W_L \nu_L, \quad l'_L = W_L l_L, \quad l'_R = W_R l_R, \quad \nu'_R = W_R \nu_R$$

transforming the lepton mass matrices as

$$M'_l = W_L^\dagger M_l W_R, \quad M'_{\nu D} = W_L^\dagger, M_{\nu D} W_L.$$

- Using this facility, the lepton mass matrices can be reduced to

$$M_l = \begin{pmatrix} C_l & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} C_{\nu D} & A_{\nu D} & 0 \\ A_{\nu D}^* & D_{\nu D} & B_{\nu D} \\ 0 & B_{\nu D}^* & E_{\nu D} \end{pmatrix}.$$

- In the language of texture zero mass matrices, these matrices are texture one zero type, together these are considered as texture two zero lepton mass matrices.

- An analysis of these mass matrices on the lines of the one carried out for the case of quarks reveals that unlike the earlier case, we find a large number of possibilities of viable texture zero lepton mass matrices.

- This is primarily due to the fact that mixing patterns and mass orderings are quite different in the two cases.

- Therefore, in the present case after establishing the viability of these mass matrices, we have examined the implications of these mass matrices for CP violation in the leptonic sector.

Numerical analysis

- Current data for neutrino mixing parameters from the latest global fits
  - Garcia et al., *Nucl. Phys. B* (2016).

Parameter	3σ range
$\Delta m^2_{\rm sol}$ $[10^{-5}\ {\rm eV}^2]$	(7.02-8.09)
$\Delta m^2_{\rm atm}$ $[10^{-3}\ {\rm eV}^2]$	(2.317-2.607)(NO); (2.590-2.307)(IO)
$\sin^2\theta_{13}$ $[10^{-2}]$	(1.86-2.50)(NO); (1.88-2.51)(IO)
$\sin^2\theta_{12}$ $[10^{-1}]$	(2.70-3.44)
$\sin^2\theta_{23}$ $[10^{-1}]$	(3.82-6.43)(NO); (3.89-6.44)(IO)

- Further, for ready reference, we also present the following 3σC.L. ranges of the PMNS matrix elements.

$$U_{\rm PMNS} = \begin{pmatrix} 0.801-0.845 & 0.514-0.580 & 0.137-0.158 \\ 0.225-0.517 & 0.441-0.699 & 0.614-0.793 \\ 0.246-0.529 & 0.464-0.713 & 0.590-0.776 \end{pmatrix}$$

- While carrying out our analysis, the magnitudes of solar and atmospheric neutrino mass square differences, defined respectively as $m_2^2 - m_1^2$ and $m_3^2 - (m_1^2 + m_2^2)/2$, are given variation within their 3σ ranges. The lightest neutrino mass, m_1 for the case of NO and m_3 for the case of IO, is considered as a free parameter while the other two masses are obtained as:

$$\mathrm{NH}: \quad m_2^2 = \Delta m_{sol}^2 + m_1^2, \quad m_3^2 = \Delta m_{atm}^2 + \frac{(m_1^2 + m_2^2)}{2},$$

$$\mathrm{IH}: \quad m_2^2 = \frac{2(m_3^2 + \Delta m_{atm}^2) + \Delta m_{sol}^2}{2}, \quad m_1^2 = \frac{2(m_3^2 + \Delta m_{atm}^2) - \Delta m_{sol}^2}{2}.$$

- For both the mass orderings of neutrinos, the range of the lightest neutrino mass has been explored from 10^{-8}eV $- 10^{-1}$eV, our conclusions remain unaffected even if the range is extended further. The phases Φ_1, Φ_2 have been considered to be free parameters and given full variation from 0 to 2π. Further, the mass matrix elements $D_{l,\nu D}$ and $C_{l,\nu D}$ have also been considered as free parameters, however, these have been constrained such that diagonalizing transformations O_l and O_ν always remain real.

Texture two zero mass matrices

- We have presented the results for the normal and inverted ordering of neutrino masses, those for the degenerate scenario can be derived from these.

Inverted mass ordering

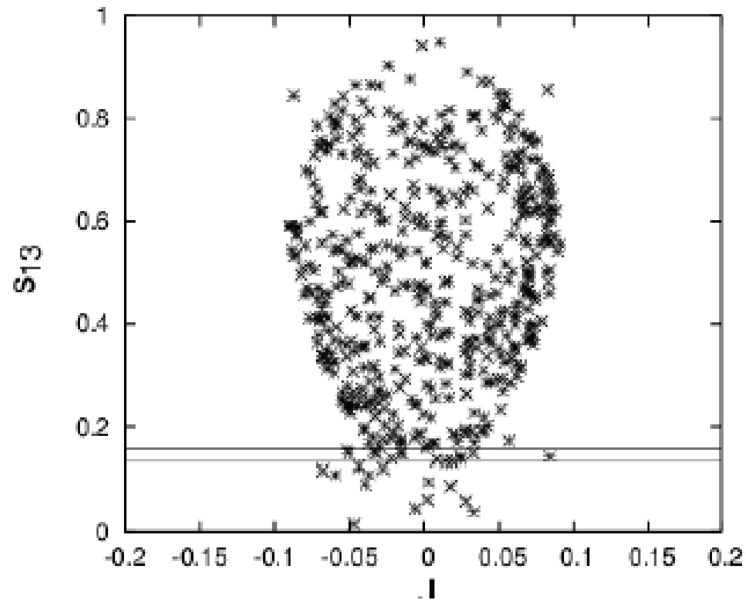
- For the inverted mass ordering, to begin with, for the mass matrices, we attempt to find the magnitudes of the PMNS matrix elements

$$U^{IH}_{PMNS} = \begin{pmatrix} 0.031 - 0.861 & 0.0868 - 0.591 & 0.132 - 0.995 \\ 0.254 - 0.977 & 0.065 - 0.815 & 0.046 - 0.802 \\ 0.105 - 0.622 & 0.392 - 0.819 & 0.084 - 0.806 \end{pmatrix}.$$

- The 3σC.L. ranges of the PMNS matrix elements given by Garcia et al. are inclusive in the ranges of the PMNS matrix elements found here, thereby ensuring the viability of texture two zero mass matrices for the inverted hierarchy case.

- The constraints for the CP violating Jarlskog's rephasing invariant parameter in the leptonic sector J_l can be obtained using the relation between the mixing matrix elements and the parameter J_l

- We get the parameter J_l versus the mixing angle s_{13} plot.



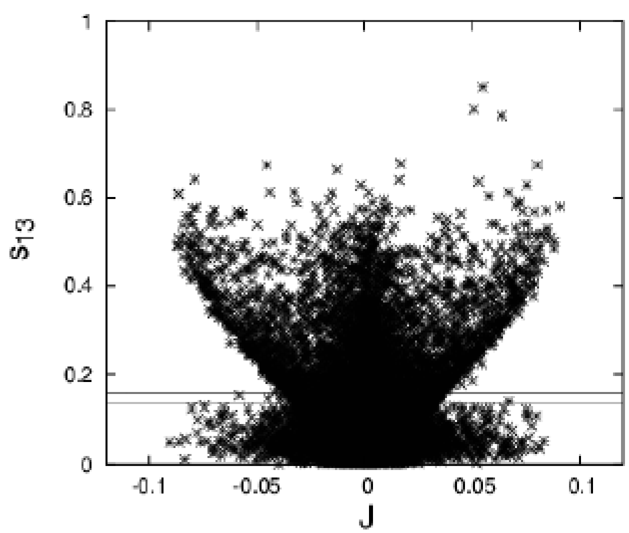
- Interestingly, one obtains a range of ~—0.05-0.05for the parameter $J_{l.}$

Normal mass ordering

- Compatibility is again established by reproducing the PMNS matrix elements.

$$U^{NH}_{PMNS} = \begin{pmatrix} 0.440-0.991 & 0.119-0.841 & 0.005-0.290 \\ 0.065-0.812 & 0.407-0.936 & 0.042-0.876 \\ 0.017-0.842 & 0.051-0.783 & 0.459-0.991 \end{pmatrix}.$$

- From the J_l versus the mixing angle s_{13} plot,



We get a range $J_l \sim -0.03 - 0.03$.

Texture four zero mass matrices

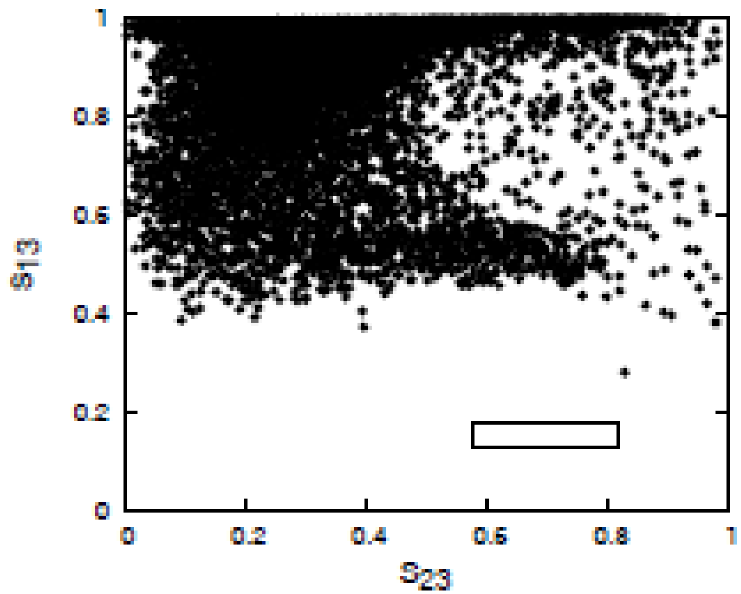
- Keeping in mind the quark lepton unification as well as required by most of the GUTs, it therefore becomes interesting to investigate the implications of similar type of mass matrices in the leptonic sector as well.

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} 0 & A_{\nu D} & 0 \\ A_{\nu D}^* & D_{\nu D} & B_{\nu D} \\ 0 & B_{\nu D}^* & E_{\nu D} \end{pmatrix}.$$

Inverted mass ordering

- Interestingly, the present well defined data rules out the inverted hierarchy of neutrino masses as the PMNS matrix constructed using the mass matrices is not compatible with the one presented by Garcia et al..

- The plot showing the parameter space corresponding to the mixing angles s_{13} and s_{23} confirms this.



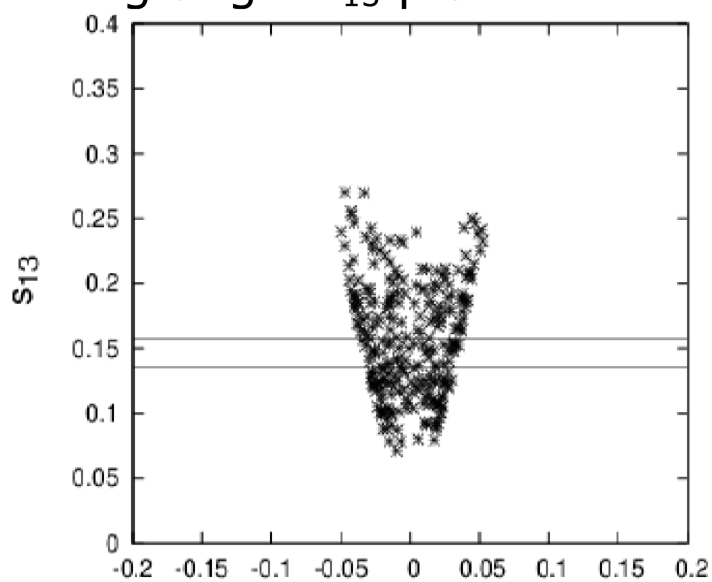
- The blank rectangular region indicates the experimentally allowed 3σregion of the plotted angles. The graph clearly shows that the plotted parameter space does not include simultaneously the experimental bounds of the plotted angles.

Normal mass ordering

- A comparison of the PMNS matrix found here with the one given by Garcia et al establishes the viability.

$$U_{PMNS}^{NH} = \begin{pmatrix} 0.696-0.995 & 0.076-0.708 & 0.031-0.199 \\ 0.072-0.699 & 0.414-0.891 & 0.182-0.822 \\ 0.052-0.590 & 0.162-0.753 & 0.555-0.974 \end{pmatrix}.$$

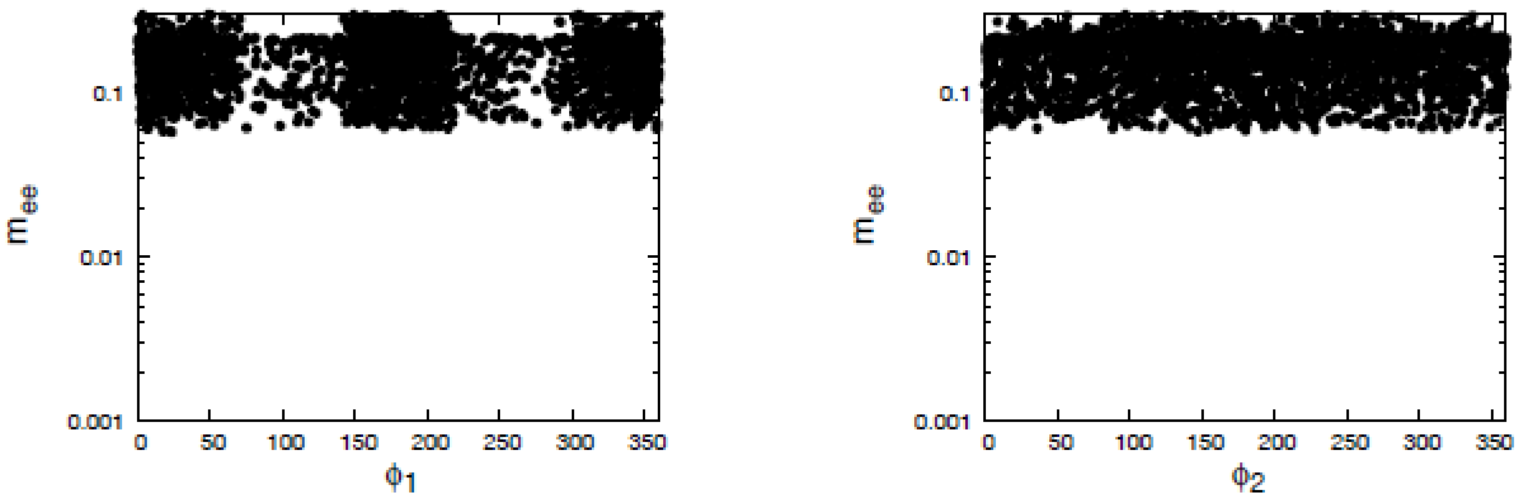
- The J_l versus the mixing angle s_{13} plot.



Indicates a range of ~—0.03-0.03 for the parameter J_l.

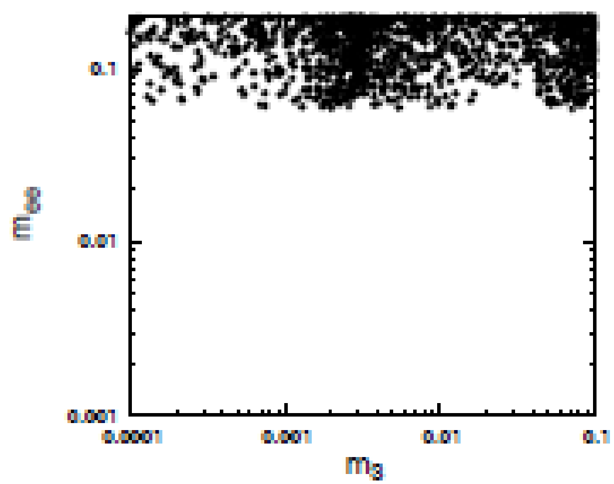
Implications of general mass matrices on the effective Majorana mass m_{ee}

- After ensuring the viability of texture two zero mass matrices for the inverted hierarchy case, we examine the constraints obtained for the parameter m_{ee}.
- To this end, we present the plots showing mass m_{ee} versus the phases Φ_1 , Φ_2, these being related to the phases of the mass matrices.



- While plotting these figures, all the three mixing angles have been constrained by their 3σexperimentalbounds, while the Majorana phases $\eta_1$and $\eta_2$as well as the other free parameters have been allowed full variation.

- It is immediately clear from the graphs that we obtain a lower bound of the order of 0.08 eV on m_{ee}, independent of the values of the phases $\Phi_1$and Φ_2. Interestingly, this bound is tantalizingly close to the likely explored range of m_{ee} by the ongoing experiments. Therefore, an absence of a signal of NDBD by these experiments would have important implications for the inverted neutrino mass ordering scenario.
- Further, to examine the dependence of parameter m_{ee} on the lightest neutrino mass m_3, we have presented m_{ee} versus m_3, plotted by giving full variation to other parameters.



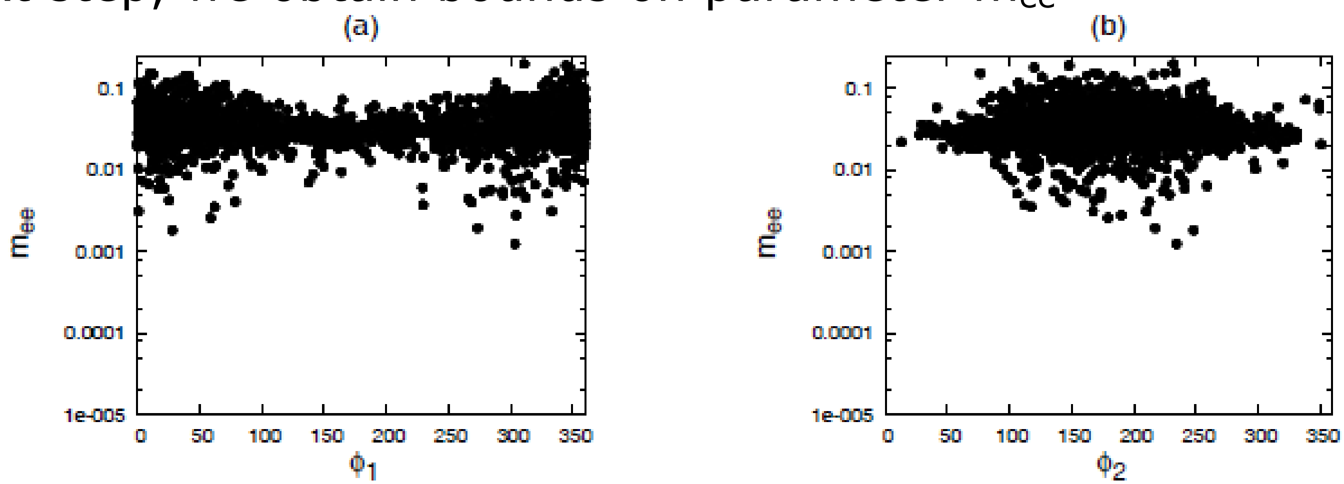
- From the graph, one finds that the above mentioned bound on parameter m_{ee} looks to be independent of the range of mass $m_3$considered here.

Normal mass ordering

- Compatibility established by reproducing the PMNS matrix elements.

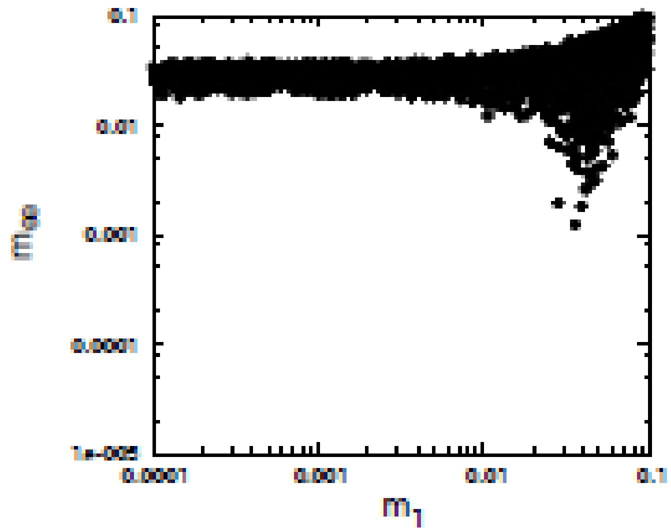
$$U_{PMNS}^{NH} = \begin{pmatrix} 0.440 - 0.991 & 0.119 - 0.841 & 0.005 - 0.290 \\ 0.065 - 0.812 & 0.407 - 0.936 & 0.042 - 0.876 \\ 0.017 - 0.842 & 0.051 - 0.783 & 0.459 - 0.991 \end{pmatrix}$$

- As a next step, we obtain bounds on parameter m_{ee}



- The parameter m_{ee}, contrary to the IO case, shows substantial dependence on phases $\Phi_1$and Φ_2. One now obtains a lower bound around 0.001 eV for m_{ee}, this being considerably lower compared with the bound obtained for IO case.

- Further, we study the variation of m_{ee} w.r.t. the lightest neutrino mass m_1.

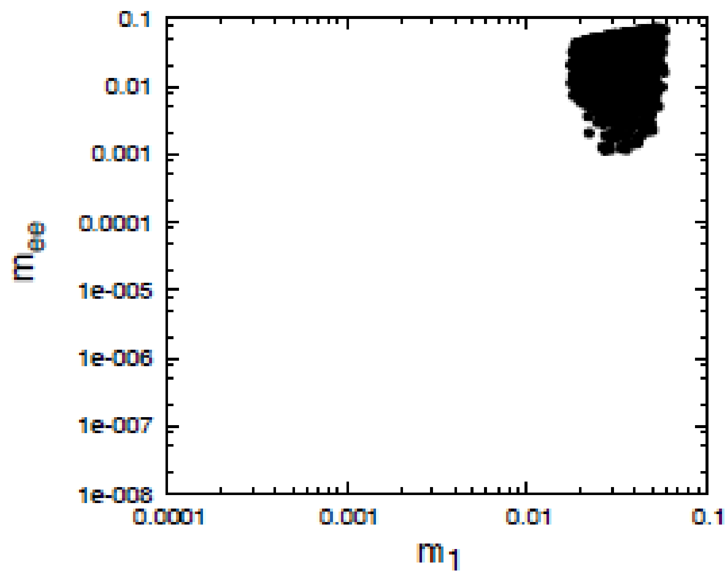


- For m_1 from 0.0001 eV−0.01 eV, the bound on parameter mee gets further sharpened and one obtains m_{ee} within the band 0.014 - 0.042 eV, whereas for m_1>0.01 eV, the parameter m_{ee} does not remain constrained to the abovementioned band but instead there is a considerable spreading of the m_{ee} values outside the band.
- In case the range of parameter m_{ee} settles around values outside the band, which is possible in the near future as several ongoing experiments like GERDA, CUORE, MAJORANA and EXO are already aiming to approach sensitivity on m_{ee} around these values, then the allowed range of m_1 would correspond to the degenerate scenario of neutrino masses.

Texture four zero mass matrices

- Regarding the predictions for bounds on the parameter m_{ee}, the plots of m_{ee} versus phases $\Phi_1$and $\Phi_2$yield results similar to the case of texture two zero mass matrices, i.e., one obtains a lower bound of the order of 0.001 eV.

- However, a plot depicting mass m_{ee} versus the lightest neutrino mass m_1, yield results different from those obtained in texture two zero case.



- In particular, a careful comparison of the plots showing parameter m_{ee} versus mass $m_1$for the texture two zero and texture four zero normal ordering cases reveals that in the case of latter one obtains a very limited region of viability.

- Therefore, non zero values of the (1,1) elements in the neutrino mass matrices leads to significantly different predictions as compared to the case when both of these elements are zero. This, interestingly, is contrary to the observation in the quark sector wherein the (1,1) element seems to be essentially redundant.

- In particular, the mass matrices lead to an upper bound of the order of 0.09 eV for the parameter m_{ee}. Further, the range of the lightest neutrino mass gets severely constrained too, viz. 0.02-0.08 eV.

- Therefore if, by any theoretical considerations, texture four zero structure turns out to be only viable possibility then it would be very easy to rule out or establish the Majorana nature of neutrinos within next few years.

Concluding remarks

- Within the Standard Model, starting with the most general mass matrices, we have used the facility of making weak basis transformations and have carried out their analysis within the texture-zero approach.

- For the case of quarks, interestingly, a particular set of texture-4 zero quark mass matrices can be considered to be a unique viable option for the description of quark mixing data and for accommodation of CP violation.

- For the case of leptons, we obtain interesting bounds on the parameter J_l for NO and IO cases of texture two zero and for NO case of texture four zero mass matrices.

- Using texture 2 zero lepton mass matrices, we have attempted to obtain bounds for m_{ee} and the corresponding lightest neutrino mass for different neutrino mass hierarchies. In the light of the bounds so obtained, the future experiments in this direction, are thus, expected to have important implications for determining the texture structure of lepton mass matrices.

Thank You